The Remainder & Factor Theorems

- **Remainder Thm:** if ~~function~~ polynomial $P(x)$ is divided by $(x-r)$, then the remainder is $P(r)$.

Given: $2x^3+3x-8 \div x-2$

$f(-2) = 2(+2)^3+3(+2)-8$

$= 2(+8)+6-8$

$= 16-2$

$= 14$ this is the remainder or $\frac{14}{x-2}$

- **Factor Thm:** $(x-r)$ is a factor of $P(x)$ if and only if $P(r)=0$.

Given: $2x^3-3x^2+x \div x-1$

$P(1) = 2(1)^3-3(1)^2+1$

$= 2-3+1$

$P(1) = 0$ since $P(1)=0$, means $x-1$ is a factor

* Difference between theorems Given $P(x) \div x-r$

- if $P(r) = \#$ it is a remainder of $\frac{\#}{x-r}$ not a factor.
- if $P(r) = 0$ it means $x-r$ is a factor

Synthetic Division

- this is a shortcut for dividing a polynomial by a binomial of $x-r$

ex: $x^3+4x^2-3x-5 \div x+3$

- synthetic division uses just the coefficients

- take the coefficients of the polynomial
- take opposite of binomial # (r)

$$\begin{array}{r|rrrr} -3 & 1 & 4 & -3 & -5 \\ & & -3 & -3 & 18 \\ \hline & 1 & 1 & -6 & 13 \end{array}$$

- Bring down 1st coefficient
- multiply r by #
- then add
- then repeat
- the final # is the remainder

- take #'s and rewrite as a polynomial, start one degree less than original polynomial
- taking 1 1 -6 / 13

$$x^2+x-6+\frac{13}{x+3}$$

Synthetic Division Continued

example: $x^3 - x^2 + 2 \div x+1$

$$
\begin{array}{r|rrrr}
-1 & 1 & -1 & 0 & 2 \\
 & & -1 & 2 & -2 \\
\hline
 & 1 & -2 & 2 & \big| \; 0
\end{array}
$$

* notice in the poly there is no x, $\therefore$ place a zero in that place

* anytime missing a degree place zero as place holder

Answer: $x^2 - 2x + 2$ with remainder of 0

* since remainder is 0, that means $x+1$ is a factor of the polynomial.

* when a polynomial is divided by one of its binomial factors $x-r$, the quotient is called a depressed polynomial.

example: $2x^3 - 3x^2 + x \div x - 1$

$$
\begin{array}{r|rrr}
1 & 2 & -3 & 1 \\
 & & 2 & -1 \\
\hline
 & 2 & -1 & \big| \; 0
\end{array}
$$

$\therefore$ the zero's of this function are $-1, 0, \frac{1}{2}$

$2x^2 - 1x$

* can be rewritten as $x(2x-1)$; the other factors can be found:

$x = 0 ; \frac{1}{2}$

Synthetic Cont...

- Synthetic can be used to find factors of the polynomial function

Ex: Determine the binomial factors of $x^3 - 7x + 6$

- since Degree of 3 means 3 complex roots

* choose a factor of 6 to try

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

← since zero $x-1$ is a factor

$x^2 + x - 6$ is left

factor $(x+3)(x-2)$

the factors are $(x+3), (x-2), (x-1)$

Ex: $x^3 + 3x^2 + 3x + 1$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

← means factor

$x^2 + 2x + 1$

$(x+1)(x+1)$

∴ factors are $(x+1), (x+1), (x+1)$