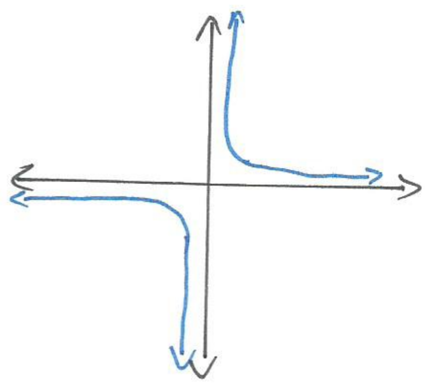
Rational Functions

Def: a rational function is a quotient of two polynomials ~~functions~~. It has the form of $f(x) = \frac{g(x)}{h(x)}$, where $h(x) \neq 0$.

parent function is $f(x) = \frac{1}{x}$

Graph:



* the graph has two branches which both branches approach asymptotes:

(lines that graphs approach but never intersect)

* there are two asymptotes; vertical & horizonatal
  - vertical are form $x =$
  - horizontal are form $y =$

for $f(x) = \frac{1}{x}$, asymptotes are $x = 0$ (y-axis) & $y = 0$ (x-axis)

* the vertical asymptote is derived from what makes the denominator zero.

* there are multiple methods to derive the horizontal asymptotes.

Rational Functions Cont...

* To Determine horizontal asymptotes

– Compare the degree of the numerator with the degree of the denominator.

① If the degree's are the same, then take the ratio of the coefficients.

ex: $f(x) = \dfrac{2x+1}{x-4}$

vertical asymptote:

$x - 4 = 0$
$\underline{+4 \quad +4}$
$x = 4$

- degree of the numerator $= 1$
- degree of the denominator $= 1$
- ratio of coefficients : $\frac{2}{1}$ $\therefore$ $y = 2$ horizontal asymp.

② If the degree is higher in the denominator, then the asymptote is $y=0$ or the vertical shift if there is one.

ex: $f(x) = \dfrac{x}{x^2-1}$

vertical asymp: $x = 1$; $x = -1$

horizontal asymp: $y = 0$

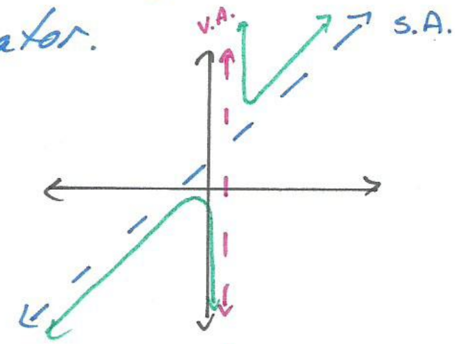
$f(x) = \dfrac{1}{x+4} - 2$

V.A. $x = -4$

H.A. $y = -2$

Asymptotes Cont...

③ there is a third type of asymptote known as a slant asymptote. A slant asymptote occurs when the degree of the numerator is exactly one higher than the degree of the denominator.



ex: $f(x) = \dfrac{x^2+4}{x-1}$

V.A. $\Rightarrow$ $x = 1$

since the degree is higher, there is no horizontal asymptote, there is a slant asymptote

Examples: Determine the asymptotes for the rational function

① $f(x) = \dfrac{x^3+1}{2x-1}$

V.A. $x = \frac{1}{2}$

&

slant asymp.

② $f(x) = \dfrac{4x+1}{8x+2}$

V.A. $x = \frac{-1}{4}$

h.A. $y = \frac{1}{2}$

③ $f(x) = \dfrac{2x}{x^2-9}$

V.A. $x = 3$ & $x = -3$

h.A. $y = 0$

④ $f(x) = \dfrac{3x^2+2}{2x^2-8}$

V.A $\Rightarrow$ $x = 2$ & $x = -2$

h.A. $y = \frac{3}{2}$

⑤ $f(x) = \dfrac{2}{x+5} + 2$

V.A $x = -5$

h.A. $y = 2$