

Rational Equations

Example 1:
$$\frac{x}{1} + \frac{x^2-5}{x^2-1} = \frac{x^2+x+2}{x+1}$$

$(x+1)(x-1)$

Key:

① make sure all denominators are simplified

LCD: $(x-1)(x+1)$

Res: $x \neq 1, -1$

Sol: $x = 3, \cancel{-1}$

② Find LCD

③ Determine the restrictions

④ multiply each term by LCD

$$\frac{x}{1} + \frac{x^2-5}{x^2-1} = \frac{x^2+x+2}{x+1}$$

~~$(x+1)(x-1)$~~ ~~$(x+1)(x-1)$~~ ~~$(x+1)(x-1)$~~

$$x(x^2-1) + x^2-5 = (x-1)(x^2+x+2)$$

$$\cancel{x} - \cancel{x} + \frac{x^2}{2} - 5 = \cancel{x} + \frac{x}{2} - 2$$

$$x - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

Example:
$$\frac{x+4}{x} + \frac{3}{x-4} = \frac{-16}{x^2-4x}$$

$x(x-4)$

LCD: $x(x-4)$

Res: $x \neq 0, 4$

Sol: $x = \cancel{1}, -3$

$$\frac{x+4}{x} + \frac{3}{x-4} = \frac{-16}{x(x-4)}$$

$$(x+4)(x-4) + 3x = -16$$

$$\cancel{x^2-16} + 3x = \cancel{-16}$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = 0, -3$$

$$x = 0$$

$$x+3=0$$

Example:

$$\frac{1}{6} = 1 + \frac{n+2}{6n}$$

LCD: $6n$

Res: $n \neq 0$ $R = 6n + n + 2$

sol: $n = -\frac{1}{3}$ $n = 7n + 2$

$$-6n = 2$$

$$n = -\frac{1}{3}$$

Example:

$$\frac{1}{x} + \frac{1}{x+4} = \frac{5x+10}{x^2+4x}$$
$$x(x+4)$$

LCD: $x(x+4)$

Res: $x \neq 0, -4$

sol: $x = -2$

$$x+4 + x = 5x+10$$

$$2x+4 = 5x+10$$

$$\underline{-2x-10} \quad \underline{-2x-10}$$

$$-6 = 3x$$

$$-2 = x$$