§ Polynomial Functions

- polynomial ⇒ a multi term expression
- Degree of a polynomial ⇒ the greatest exponent of the variable
- Leading coefficient ⇒ the coefficient of the variable with the greatest exponent.
- Polynomial function ⇒ function where $f(x)$ = a polynomial expression
- if $f(x)$ is a polynomial function, then it has <u>zeros</u> of the function. (zeros of a function is where graph crosses x-axis)
  - a <u>zero</u> of a function is any value of x that makes the function = zero. ∴ $f(x) = 0$
- if a value is a <u>zero</u> of a <u>function</u>, it is also the <u>solution</u> for the <u>polynomial equation</u>
  - solutions for equations are known as roots.
- zero & root are often used interchangeably, but <u>functions</u> have <u>zeros</u> & <u>equations</u> have <u>roots</u>
- zero's & roots fall up under the set of <u>Complex #'s</u>
  - complex #s are of the form: $a + bi$, which is the combination of a real # & imaginary
  - a pure imaginary # is just 'bi':
    - imaginary a derived from the $\sqrt{-1} = i$ also $i^2 = -1$

* only real # roots/zeros cross the x-axis

* The Fundamental Thm of Algebra

- if a function has a degree > 0, than the function has at least <u>one</u> <u>root</u> in the complex # set.

- <u>corollary</u> $\Rightarrow$ the degree 'n' of an equation means there a 'n' number of complex roots

<u>Examples:</u> Determine roots & leading coefficient.

① $f(x) = x^3 - 6x^2 + 10x - 8$

- degree is 3
- 3 complex roots
- leading coefficient is 1

② $f(x) = 5x^2 + 8x^5 - 2$

- degree of 5
- 5 complex roots
- leading coefficient is 8

<u>Examples:</u> Determine whether each # is a root.

③ $x^3 - 5x^2 + 3x - 18 = 0$; 5

$5^3 - 5(5)^2 + 3(5) - 18 = 0$

$-3 = 0$

- not a root

④ $f(x) = x^3 - 6x^2 + 10x - 8$; 4

$f(4) = 4^3 - 6(4)^2 + 10(4) - 8$

$f(4) = 0$

- 4 is a root

Examples: state # of roots ; find the roots

⑤ $f(x) = 9x^4 - 35x^2 - 4 =$

- 4 complex roots

$9x^4 - 35x^2 - 4 = 0$

$(9x^2+1)(x^2-4) = 0$

$9x^2 + 1 = 0 \qquad x^2 - 4 = 0$

$9x^2 = -1 \qquad x^2 = 4$

$x^2 = \frac{-1}{9} \qquad \boxed{x = \pm 2}$

$\boxed{x = \pm \frac{1}{3} i}$ ← roots

⑥ $f(x) = x^3 + 2x^2 - 8x$

- 3 complex roots

$x^3 + 2x^2 - 8x = 0$

$x(x^2 + 2x - 8) = 0$

$x(x+4)(x-2) = 0$

$x = 0, \quad x + 4 = 0, \quad x - 2 = 0$

$\boxed{x = 0, -4, 2}$ roots

Examples: Write an equation given the roots ; tell how many times the function crosses x-axis?

⑦ $\cancel{f(x)} =$ 4, 3, −2

- 3 real roots means crosses x-axis <u>3</u> times

Eq: $(x-4)(x-3)(x-2)$

$(x^2 - 3x - 4x + 12)(x-2)$

$(x^2 - 7x + 12)(x-2)$

$$\left.\begin{array}{l} x^3 - 7x^2 - 12x \\ -2x^2 + 14x - 24 \end{array}\right\} x^3 - 9x^2 + 2x - 24$$

Equation

⑧ 3, −2, 4i, −4i

- only 2 real roots, crosses 2 times

$\underbrace{(x-3)(x+2)}_{x^2 + 2x - 3x - 6}\ \underbrace{(x-4i)(x+4i)}_{x^2 - 4ix + 4ix - 16i^2}$ $\quad i^2 = -1$

$\underbrace{(x^2 - x - 6)\ (x^2 + 16)}$

$$\begin{array}{r} x^4 - x^3 - 6x^2 \\ + 16x^2 - 16x - 96 \\ \hline \end{array}$$

$y = x^4 - x^3 + 10x^2 - 16x - 96$

§ Quadratic Equations

- polynomial equation with a degree of 2
- has 2 complex roots

Methods for finding roots of a Quadratic

① factoring

② Completing the square

③ Quadratic Formula

① Factoring - factoring quadratic into ( )

ex: $x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x - 8 = 0 \qquad x + 2 = 0$

$\boxed{x = +8 \qquad x = -2}$

roots

ex: $x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x - 7 = 0 \qquad x - 3 = 0$

$\boxed{x = 7 \ ; \ x = 3}$

roots

② Completing the Square

- looking for a value that makes one side a perfect square

- Perfect squares: $25 \to 5 \times 5$ $\quad (x+4)^2 \to (x+4) \times (x+4)$

Given: $x^2+6x+\underline{\ 9\ }$

$(x+3)(x+3)$

what can you add to the expression that would give you the same # in the ( ).

- adding 9 to the expression allows you to factor into the same ( ), thus completing a perfect square.

Given: $x^2+10x+\underline{\ 25\ }$

$(x+5)(x+5)$

$= (x+5)^2$

Given: $x^2+3x+\underline{\ \frac{9}{4}\ }$

$(x+\frac{3}{2})(x+\frac{3}{2})$

$= (x+\frac{3}{2})^2$

* notice that the value in the ( ) is half of the x coefficient.

- $\frac{10}{2} = 5$
- $\frac{3}{2} = \frac{3}{2}$

Completing the Square Cont...

thus to complete square

① leading coefficient **must** be 1

② divide x coefficient by 2

ex: $x^2 - 12x + \underline{36}$

$\frac{-12}{2} = -6$ $\quad (-6)^2$

$\therefore (x-6)^2$

$x^2 - 11x + \underline{\frac{121}{4}}$

$\frac{-11}{2} = \frac{-11}{2}$ $\quad \left(\frac{-11}{2}\right)^2$

$\therefore \left(x - \frac{11}{2}\right)^2$

to solve by completing square

ex: ① $x^2 + 4x = 12$

$x^2 + 4x + \underline{4} = 12 + 4$

$\sqrt{(x+2)^2} = \sqrt{16}$

$x + 2 = \pm 4$

$x + 2 = 4$; $x + 2 = -4$

$x = 2$; $x = -6$

* since you add 4 to the left to complete square, you must add 4 to right to keep balanced.

ex: ② $x^2 - 3x = 4$

$x^2 - 3x + \underline{\frac{9}{4}} = 4 + \frac{9}{4}$

$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{25}{4}}$

$x - \frac{3}{2} = \pm \frac{5}{2}$

$x - \frac{3}{2} = \frac{5}{2}$; $x - \frac{3}{2} = \frac{-5}{2}$

$x = 4$; $x = -1$

③ The Quadratic Formula

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ based from $y = ax^2 + bx + c$

- the quadratic Formula can be used to solve any quadratic

- the inside of the radical b^2-4ac is known as the discriminant.
- the discriminant describes what type of roots the quadratic will have
  - if $b^2-4ac > 0$, there are 2 real sol. (roots)
  - if $b^2-4ac = 0$, there is 1 unique real sol. (roots)
  - if $b^2-4ac < 0$, there are 2 imaginary sol. (roots)

Examples: Find the discriminant & describe roots

① $\overset{a}{2x^2} \overset{b}{- 3x} \overset{c}{+ 4} = 0$

$b^2 - 4ac$

- $(-3)^2 - 4(2)(4)$

$9 - 32$

$= -23$

2 imaginary roots

② $3x^2 + 9x^2 + 2 = 0$

$b^2 - 4ac$

- $9^2 - 4(3)(2)$

$81 - 24$

$= 57$

2 real sol. (roots)

Examples: Find roots using Quadratic Formula

① $2x^2 - 6x + 5 = 0$

$(-6)^2 - 4(2)(5)$

$36 - 40 = -4$ 2 imag roots

$= \dfrac{-(-6) \pm \sqrt{-4}}{2(2)}$

$= \dfrac{6 \pm 2i}{4} = \dfrac{3 \pm i}{2}$

② $x^2 + 5x - 36 = 0$

$\dfrac{-5 \pm \sqrt{5^2 - 4(1)(-36)}}{2(1)}$

$= \dfrac{-5 \pm \sqrt{169}}{2}$ 2 real roots

$= \dfrac{-5 \pm 13}{2}$

$= \dfrac{-5+13}{2}$; $\dfrac{-5-13}{2}$

$\boxed{= 4 \ ; \ -9}$

Examples: Completing the square with leading coefficient other than 1.

- $3x^2 + 7x + 7 = 0$
- $\dfrac{3x^2}{3} + \dfrac{7x}{3} + \dfrac{7}{3} = 0$ 1st: Divided each term by the leading coefficient
- $x^2 + \dfrac{7}{3}x + \dfrac{7}{3} = 0$
- $x^2 + \dfrac{7}{3}x + \dfrac{49}{36} = \dfrac{-7}{3} + \dfrac{49}{36}$ • take half of $\dfrac{7}{3}$
- $\sqrt{\left(x + \dfrac{7}{6}\right)^2} = \sqrt{\dfrac{-35}{36}}$ • take square root
- $x + \dfrac{7}{6} = \pm i \dfrac{\sqrt{35}}{6}$
- $x = \dfrac{-7}{6} \pm i \dfrac{\sqrt{35}}{6}$ or $\dfrac{-7 \pm i\sqrt{35}}{6}$

The Quadratic Formula

from standard form $y = ax^2 + bx + c$

$$ax^2 + bx + c = 0$$

- complete the square

1st $$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

2nd $$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

3rd $$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

4th $$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

5th $$\left(x + \frac{b}{2a}\right)^2 = \frac{4a}{4a}\frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

6th $$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

7th $$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

* the Quadratic Formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$