Piecewise Functions

"Morgan's" def: functions with different pieces or multiple parts that are restricted by domain.

example:

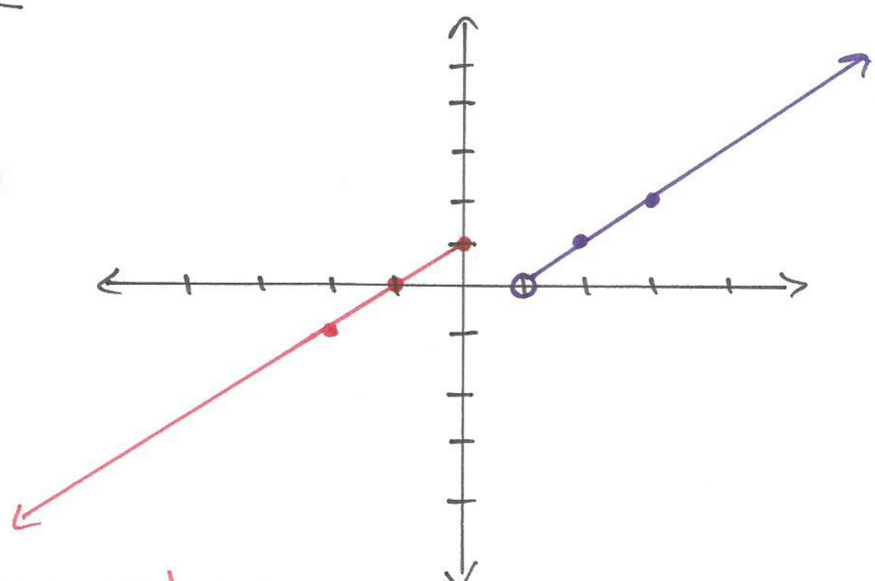
$$f(x) = \begin{cases} x+1 & \text{if } x \le 0 \\ x-1 & \text{if } x > 1 \end{cases}$$

* notice within f(x), there are two equations.
* the equation used is based on the domain.
  - $x+1$ is used when $x \le 0$
  - $x-1$ is used when $x > 1$

Graph:

$x+1 \quad x \le 0$

x	y
0	1
-1	0
-2	-1



$x-1 \quad x > 1$

x	y
1	0
2	1
3	2

* the line continues going down to the left.
* notice at (0,1) it is a closed dot b/c of $\le$

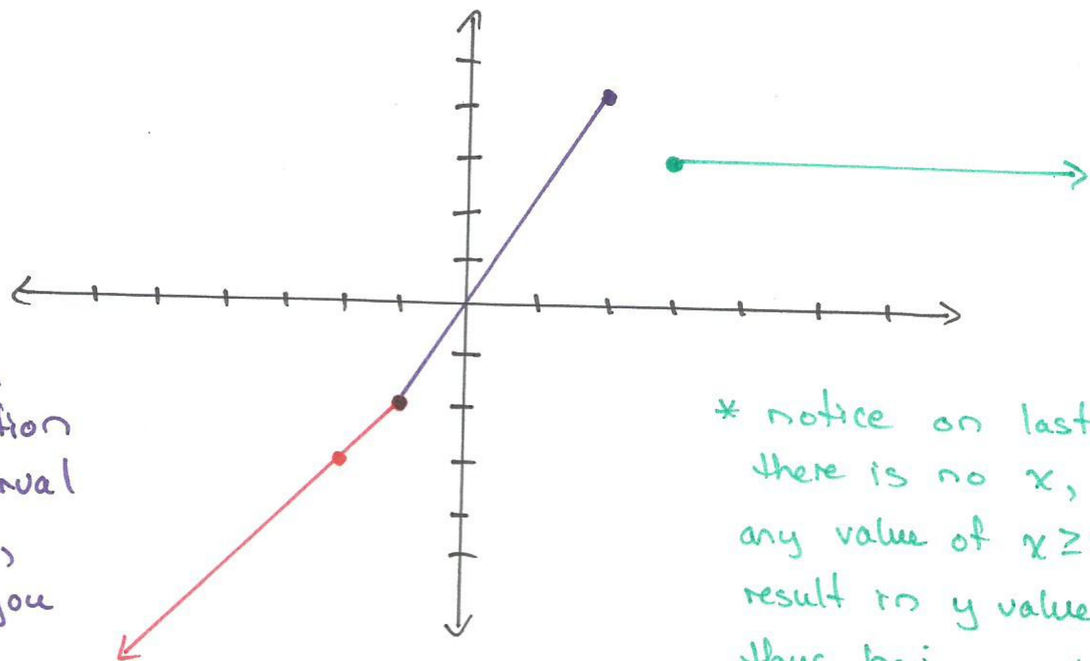
* the line continues up to the right.
* notice at (1,0) it is an open dot b/c of $>$

Piecewise Cont...

Example 2

$$f(x) = \begin{cases} x-1, & \text{if } x \le -1 \\ 2x, & \text{if } -1 \le x \le 2 \\ 3, & \text{if } x \ge 3 \end{cases}$$

* since 3 equations, means 3 graphs



* notice on middle equation you have interval b/w $-1 \le x \le 2$, this means you have a line segment.

* it is possible for graphs to share points.

* notice on last equation, there is no x, therefore any value of $x \ge 3$ will result in y value of 3, thus having a horizontal line.

* From the graphs, we also determine domain & range

Domain: what x values, start from left & work right

D: $(-\infty, 2] \cup [3, \infty)$ notice space b/w 2 & 3 on x-axis

Range: what y values, start from bottom & work up.

R: $(-\infty, 4]$

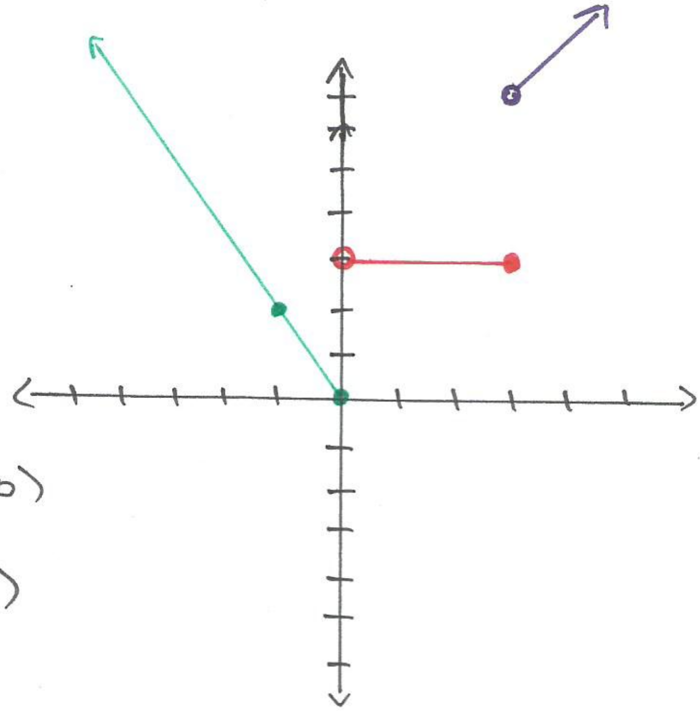
• uses [ ]

o uses ( )

Piecewise Cont...

Example 3:

$$f(x) = \begin{cases} -2x, & \text{if } x \le 0 & \text{*Line} \\ 3, & \text{if } 0 < x \le 3 & \text{*segment} \\ 2x+1, & \text{if } x > 3 & \text{*Line} \end{cases}$$



D: $(-\infty, \infty)$

R: $[0, \infty)$

Alternate methods for Domain

Option 1: from restrictions

- $x \le 0$ $\quad (-\infty, 0]$
- $0 < x \le 3$ $\quad (0, 3]$
- $x > 3$ $\quad (3, \infty)$

* notice similarities

$\therefore (-\infty, \infty)$

Option 2: push all lines flat into one horizontal line & you will get

Line goes on both ways

$\therefore (-\infty, \infty)$

Alternate method for Range:

push all lines together vertically

end result that starts at 0 & goes to infinity

$\therefore [0, \infty)$