Infinite Sequences & Series

- An <u>infinite</u> <u>sequence</u> is a sequence with an infinite # of terms.

- infinite sequences have limits, meaning as the # of terms increases the sequence is heading in a specific direction.

written as $\underbrace{\lim_{n \to \infty}}_{\text{as \# of terms heads towards } \infty} (\text{seq}) = \# \longleftarrow$ where the sequence is going.

the sequence

<u>example:</u> $\lim_{n \to \infty} \frac{1}{n} = 0$ meaning the larger the denominator gets, the closer to zero the sequence goes.

3 cases of limits

① degree larger in denominator means $\lim_{n \to \infty} = 0$

② degrees are same in num & denom means $\lim_{n \to \infty}$ = ratio of leading coefficents

③ degree larger in numerator means $\frac{1}{n \to \infty}$ = Does not exist.

Infinite cont...

Limit examples:

① $\lim\limits_{n\to\infty} \frac{5-n^2}{2n}$ degree is larger in Numerator

$\therefore \lim\limits_{n\to\infty} = \text{DNE}$

② $\lim\limits_{n\to\infty} \frac{3n-6}{7n}$ degrees are same $\therefore$ ratio of leading coeff.

$\therefore \lim\limits_{n\to\infty} = \frac{3}{7}$

③ $\lim\limits_{n\to\infty} \frac{1}{5^n}$ degree is larger in denominator,

$\therefore \lim\limits_{n\to\infty} = 0$

- An <u>infinite</u> <u>series</u> is the indicated sum of an infinite sequence.

Sum of an Infinite Geometric Series: if $|r|<1$, then $S = \frac{a_1}{1-r}$

* if $|r|>1$, then the sum <u>DNE</u>.

Infinite Cont...

Exam: find the sum of $21 - 3 + \frac{3}{7} - \ldots$

$r = \frac{-3}{21}$ or $\frac{-1}{7}$ since $\left|\frac{-1}{7}\right| < 1$ there is a sum

$S = \frac{a_1}{1-r}$ $\therefore$ $S = \frac{21}{1-\left(\frac{-1}{7}\right)}$

$$S = \frac{21}{\frac{8}{7}}$$

$$S = \frac{147}{8} \text{ or } 18\tfrac{3}{8}$$

- A <u>repeating</u> <u>decimal</u> is a decimal that infinitely repeats the same sequence of #s over & over

- a repeating decimal is actually the sum of an infinite series.

Given: $.\overline{762}$, this is actually

$$\frac{762}{1000} + \frac{762}{1{,}000{,}000} + \frac{762}{1{,}000{,}000{,}000} + \ldots$$

thus $r = \frac{1}{1000}$ & $a_1 = \frac{762}{1000}$

Using Sum of infinite series $S = \dfrac{\frac{762}{1000}}{1 - \frac{1}{1000}} = \dfrac{\frac{762}{1000}}{\frac{999}{1000}} = \dfrac{762}{999} = \dfrac{254}{333}$

$$\therefore .\overline{762} = \frac{254}{333}$$

Infinite Cont...

Ex: Given $.\overline{27}$

means $\dfrac{27}{100} + \dfrac{27}{10{,}000} + \dfrac{27}{1{,}000{,}000} + \cdots$

$r = \dfrac{1}{100} \qquad a_1 = \dfrac{27}{100}$

$$S = \frac{\frac{27}{100}}{1 - \frac{1}{100}} = \frac{\frac{27}{100}}{\frac{99}{100}} = \frac{27}{99} = \frac{9}{33}$$

Ex: Given $.3105$

$r = \dfrac{1}{10{,}000} \qquad a_1 = \dfrac{3105}{10{,}000}$

$$S = \frac{3105}{9{,}999}$$