§ Geometric Sequences & Series

- a Geometric Sequence is a sequence in which there is a constant that is multiplied between successive terms.
- the constant term is referred to as the common ratio, denoted "r".

Example: 1, 3, 9, 27 . . .

- the common ratio is 3.
- next 3 terms are 81, 243, 729

Example: 100, 50, 25, . . .

- common ratio is $\frac{1}{2}$
- next three terms would be, $\frac{25}{2}$, $\frac{25}{4}$, $\frac{25}{6}$

- To find a specific term in the Sequence

The n^{th} term of geo seq

$$a_n = a_1 r^{(n-1)}$$

- a_n: n^{th} term
- a_1: 1st term
- r: common ratio
- $(n-1)$: # of term

Geometric Cont. . .

Ex: Given: $a_n = a_1 r^{(n-1)}$; 2, 6, 18 . . . .

find 20th term

- $a_1 = 2 \quad r = 3 \quad n = 20$

$\therefore \quad a_{20} = 2 \cdot 3^{(20-1)}$

$a_{20} = 2 \cdot 1,162,261,467$

$a_{20} = 2,324,522,934$

Ex: Find a_1 given $a_5 = -6$; $r = -\frac{1}{3}$

$-6 = a_1 \cdot \left(\frac{-1}{3}\right)^{(5-1)}$

$-6 = a_1 \cdot \frac{1}{81}$

$-486 = a_1$

Geometric Cont. . .

- Geometric Mean – the terms between any two non consecutive terms of a geometric seq.

Ex: 4, 12, 36 — 12 is geometric mean of 4 & 36

2, 8, 32, 128 — 8 & 32 are two geometric means between 2 & 128

- To find geometric means.
  - find two geometric means between 3 & 81
  - means 3, —, —, 81

* you want to find "r" using $a_n = a_1 r^{n-1}$

$a_n = 81$, $a_1 = 3$, $n = 4$ since 4 terms

- $81 = 3 \cdot r^{4-1}$

$81 = 3r^3$

$27 = r^3$

$3 = r$ — knowing r use to find missing values

3, 9, 27, 81

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Geometric Cont . . .

- A geometric Series is the sum of the terms of a geometric sequence

The Sum of a finite Geometric Seqtes

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

- S_n: the sum of the first n terms
- a_1: 1st term
- n: # of terms
- r: common ratio

Ex: Given $16, -48, 144, -432, \ldots$

find the sum of first 10 terms

$$S_n = \frac{a_1(1-r^n)}{1-r} \qquad a_1 = 16 \quad n = 10$$

$$r = -3 \qquad * r = \frac{-48}{16} = -3$$

$$S_{10} = \frac{16(1-(-3)^{10})}{1-(-3)}$$

use calc.

$$S_{10} = -236,192$$