Function Symmetry

- there are multiple types of symmetry,
  point symmetry, line symmetry, symmetry with respect to the origin or y-axis or x-axis.
- we will focus on symmetry with respect to the origin & the y-axis

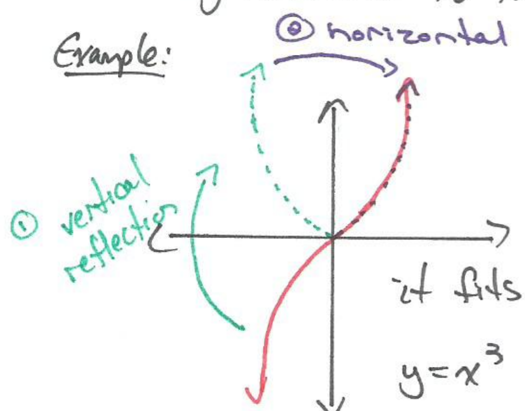
* Symmetry with respect to the origin.

- origin is at $(0,0)$

* Morgan's def of graphically being symmetric to origin:

- if you take part of the graph on one side of origin & can reflect it vertically over the x-axis & reflect it horizontally over the y-axis (both reflections), & it fits exactly on the other half of the graph, then it is symmetric to the origin.

Example:



$y = x^3$

it fits, ∴ $y=x^3$ is symmetric to origin

Algebrically Symmetric to Origin

if $f(-x) = -f(x)$, then the function is symmetric to origin

ie.

$$f(x) = x^3$$

$$f(-x) = -f(x)$$

$$(-x)^3 = -(x)^3$$

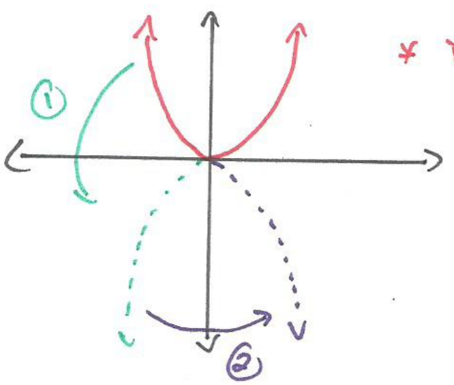
$$-x^3 = -x^3$$

same on both sides ∴ $y=x^3$ is symmetric to origin

Symmetry Cont...

* is $y=x^2$ symmetric to origin?

Graphically



* Does not fit on other half

∴ not symmetric to origin

Algebriacally

Does $f(-x) = -f(x)$

$(-x)^2 = -(x)^2$

$x^2 \neq -x^2$

* Not equal, therefore not symmetric to origin

* Differences between $y=x^3$ & $y=x^2$
  - different graphs
  - different exponents

  - key difference with the exponents is one is odd, & the other is even.

* thus to be symmetric to the origin, there must be odd exponents. ($y=x$, $y=x^3$, $y=x^5$ ...)

* functions symmetric to the origin are considered to be "Odd Functions"

* Vertical & Horizontal shifts will make a function NOT symmetric to the origin

  ex: $y=x^3+1$ is not symmetric to origin

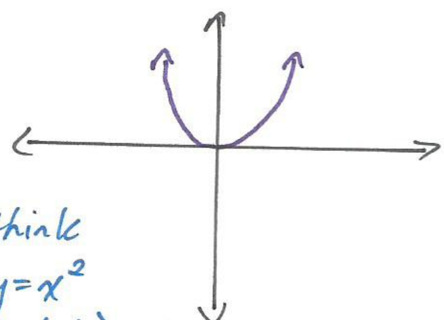
* Symmetric with respect to the y-axis

- y-axis is vertical axis

* Graphically

- means y-axis runs down the middle of your graph & divides it into two congruent pieces

i.e.



* think $y = x^2$ (parabola)
- same on both sides of y-axis

* Algebriacally

- means $f(-x) = f(x)$

using $y = x^2$

$(-x)^2 = x^2$

$x^2 = x^2$

- means plugging in negative does not change function.

* from earlier symmetric to origin has odd exponents & is an "odd function"

* symmetric to y-axis has even exponents & is considered "even function"
- one excemption is $y = |x|$

* Vertical shifts do not affect y-axis symmetry

* Horizontal shifts will make a function not symmetric to the y-axis

Examples:

Is the function symmetric to the origin, y-axis, or neither?

① $y = x^2 + 1$

y-axis

② $y = x^5$

origin

③ $y = x^5 + 4$

neither b/c of v.s.

④ $y = x^2 + x^4$

y-axis

⑤ $y = x^3 + x^7$

origin

⑥ $y = (x-4)^2$

neither b/c h.s.

⑦ $y = x^5 - x^2$

neither b/c of odd & even

⑧ $y = (x-4)^3$

neither b/c h.s.

⑨ $y = x^6 + x^4 + 1$

y-axis

⑩ $y = x^7 - x^3 + x$

origin