Decomposing Fractions

- Decomposing a fraction is the process of separating a rational fraction (expression) into the sum or difference of partial fractions

- a partial fraction is one of the fractions used to create a rational expression by adding or subtracting.

Example: Given rational fraction $\frac{7x+5}{x^2+2x-3}$

Separate into partial fractions

$\frac{7x+5}{x^2+2x-3}$, the denominator can factor into $(x+3)(x-1)$

thus,

$$\frac{7x+5}{x^2+2x-3} = \underbrace{\frac{A}{x+3} + \frac{B}{x-1}}_{\text{these will be the partial fractions}}$$

- take denominator & multiply to other numerator to create two equations.

① $7x+5 = A(x-1) + B(x+3)$ ② $7x+5 = A(x-1) + B(x+3)$

- now look at how to make the ( ) equal to 0
- thus x would be 1 & -3
- use 1 in one equation & -3 in the other

Decomposing Cont...

cont...

① $7x+5 = A(x-1) + B(x+3)$

let $x = 1$

$7(1)+5 = A(1-1) + B(1+3)$

$12 = A(0) + B(4)$

$12 = 4B$

$3 = B$

② $7x+5 = A(x-1) + B(x+3)$

let $x = -3$

$7(-3)+5 = A(-3-1) + B(-3+3)$

$-16 = A(-4) + B(0)$

$-16 = -4A$

$4 = A$

- Now substitute values for $\frac{A}{x+3}$, $\frac{B}{x-1}$

$\therefore$ fractions are $\frac{4}{x+3}$ & $\frac{3}{x-1}$

Example: $\frac{11x+21}{2x^2+9x-18}$

$$\frac{11x+21}{\underset{(2x-3)(x+6)}{2x^2+9x-18}} = \frac{A}{2x-3} + \frac{B}{x+6}$$

thus: $\frac{5}{2x-3}$

$\frac{3}{x+6}$

① $11x+21 = A(x+6) + B(2x-3)$

let $x = -6$

$11(-6)+21 = A(-6+6) + B(2(-6)-3)$

$-45 = A(0) + B(-15)$

$-45 = -15B$

$3 = B$

② $11x+21 = A(x+6) + B(2x-3)$

let $x = \frac{3}{2}$

$11\left(\frac{3}{2}\right)+21 = A\left(\frac{3}{2}+6\right) + B\left(2\cdot\frac{3}{2}-3\right)$

$\frac{33}{2}+21 = A\left(\frac{15}{2}\right) + B(0)$

$\frac{75}{2} = \frac{15}{2}A$

$5 = A$

Decomposing Cont...

ex: $\dfrac{8x+7}{x^2+x-2}$ $\quad (x+2)(x-1)$

$$\therefore \quad \frac{8x+7}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

① $8x+7 = A(x-1) + B(x+2)$

let $x=1$

$8(1)+7 = A(1-1) + B(1+2)$

$15 = 3B$

$5 = B$

② $8x+7 = A(x-1) + B(x+2)$

let $x=-2$

$8(-2)+7 = A(-2-1) + B(-2+2)$

$-9 = -3A$

$3 = A$

thus $\dfrac{3}{x+2}$ & $\dfrac{5}{x-1}$

ex: $\dfrac{3x-1}{x^2-1}$ $\quad (x-1)(x+1)$ $\quad \therefore \quad \dfrac{3x-1}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$

thus $3x-1 = A(x+1) + B(x-1)$

when $x=1$

$3(1)-1 = A(1+1)$

$2 = 2A$

$1 = A$

when $x=-1$

$3(-1)-1 = B(-1-1)$

$-4 = -2B$

$2 = B$

thus $\dfrac{1}{x-1}$ & $\dfrac{2}{x+1}$