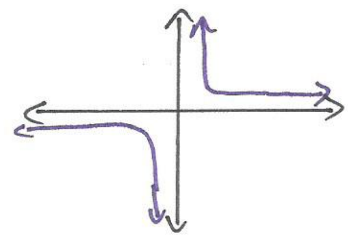
Continuity of a Function

A function is <u>continous</u> if the graph goes on & on without any break in the graph.

3 types of discontinuity

① infinite: the graph goes on & on but approaches an asymptote

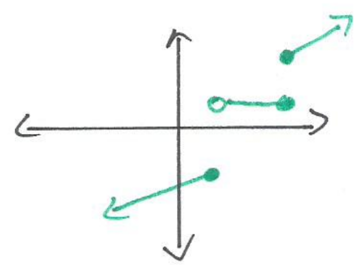
<u>ex:</u> $y = \frac{1}{x}$

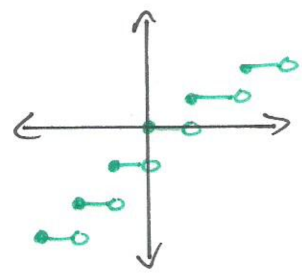


* end up with $\frac{\#}{0}$

② Jump: graph jumps from one place to another

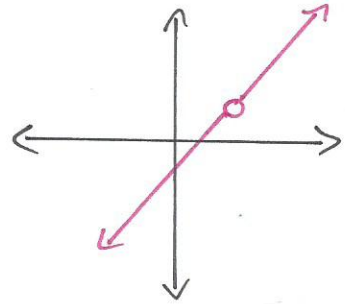
<u>ex:</u> piecewise & greater integer





③ Point: there is a hole in the graph at some point.

<u>ex:</u> $y = \frac{x^2 - 1}{x + 1}$



* end up with $\frac{0}{0}$

- with point discontinuity, there is a specific point that it is discontinous

Continuity Test

– rules that determine if a function is continous at a specific point.

given:

function $f(x)$; check if continous at $x=c$

Steps:

(1) the value of the function at $x=c$ exists. (meaning $f(c)$ is a real value)

(2) the y-value approaches the same value from the left & right side

(3) the values in step #1 & step #2 are the same value

Example: given $f(x) = 3x^2+7$; continous $x=1$

(1) Does $f(1)$ exists?

$f(1) = 3(1)^2+7$, $f(1) = 10$

value exists, therefore passes step 1.

(2) Does 'y' approach the same value from left & right at $x=1$

from left	from right
$x = .7, .8, .9$	$x = 1.1, 1.2, 1.3$
$y = 8.47, 8.92, 9.43$	$y = 10.63, 11.32, 12.07$
from left approaches 10	from right approaches 10

values are same, so passes step 2

(3) step #1 = 10 & step #2 = 10

values are same, passes step #3

* function is continous

Examples

② $f(x) = \dfrac{x-5}{x+3}$, $x = -3$

X ① $f(-3) = \dfrac{-3-5}{-3+3} = \dfrac{-8}{0}$ = value does not exists

$\therefore$ function is discontinous

failed step 1

* function has infinite discontinuity

since $\frac{\#}{0}$

③ $f(x) = \begin{cases} x+2 , & x < -2 \\ 3x , & x \geq -2 \end{cases}$ $x = -2$

✓ ① $f(-2) = -2+2$ $f(-2) = 3(-2)$

$= 0$ $= -6$

* value exists in both equations, therefore passes step 1

X ② from left $x < -2$ from right $x \geq -2$

use $x+2$ use $3x$

= y approaches 0 = y approaches −6

y approaches different values from left & right , $\therefore$ fails step #2

* this is jump discontinuity

Examples Cont...

④ $f(x) = \dfrac{x^2-4}{x-2}$, $x = 2$

X ① $f(2) = \dfrac{2^2-4}{2-2} = \dfrac{0}{0}$, value does not exists
$\therefore$ fails step #1

* since $\frac{0}{0}$, it is <u>point</u> discontinuity

to determine actual point

- simplify equation

$$\frac{x^2-4}{x-2} \Rightarrow \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} \Rightarrow x+2$$

- plug in $x = 2$

$$2+2 = 4$$

- therefore point of discontinuity is $\overset{x,y}{(2, 4)}$

<u>You Try:</u>

Ⓐ $y = x^2 - x$, $x = 1$

Ⓑ $y = \begin{cases} x+1 , & x \leq 1 \\ 2x , & x > 1 \end{cases}$, $x = 1$

Ⓒ $f(x) = \dfrac{x+3}{x^2-9}$, $x = 3$

Ⓓ $f(x) = \dfrac{x^2-2x}{x-2}$, $x = 2$