Combinations & Compositions of functions

Combination of functions:

Def: combining functions through the basic operations $(+, -, \cdot, \div)$

Notation given: $f(x)$ & $g(x)$

$f(x) + g(x) \iff (f+g)(x)$

$f(x) - g(x) \iff (f-g)(x)$

$f(x) \cdot g(x) \iff (f \cdot g)(x)$

$f(x) \div g(x) \iff \left(\frac{f}{g}\right)(x)$

$\frac{f(x)}{g(x)}$

Combine functions & state the domain

Ex: $f(x) = x^2 + 3x + 2 \qquad g(x) = 2x^2 - 9$

- $f(x) + g(x) = x^2 + 3x + 2 + 2x^2 - 9$

$$\boxed{= 3x^2 + 3x - 7}$$

D: $(-\infty, \infty)$

Ex: $h(x) = \frac{3+7x}{2x} \qquad j(x) = 4x+1$

- $h(x) - j(x) = \frac{3+7x}{2x} - \frac{4x+1}{1}\frac{(2x)}{(2x)}$

$$\frac{3+7x-(8x^2+2x)}{2x}$$

$2x = 0$

$$\frac{3+7x-8x^2-2x}{2x}$$

D: $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

$$\boxed{= \frac{-8x^2+5x+3}{2x}}$$

Given: $f(x) = \dfrac{x}{x+3}$ $\quad g(x) = x^2 - 9$

$$f(x) + g(x) \qquad \frac{x}{x+3} + \frac{x^2-9}{1} \cdot \frac{(x+3)}{(x+3)}$$

$(x^2-9)(x+3)$

$$= \frac{x + x^3 + 3x^2 - 9x - 27}{x+3}$$

$x^3+3x^2-9x-27$

$$f(x)+g(x) \; \boxed{= \frac{x^3+3x^2-8x-27}{x+3}}$$

$x \neq -3$ D: $(-\infty,-3) \cup (-3,\infty)$

$$f(x) - g(x) \qquad \frac{x}{x+3} - \frac{x^2-9}{1} \cdot \frac{(x+3)}{(x+3)}$$

$$\frac{x - (x^3+3x^2-9x-27)}{x+3}$$

D: $(-\infty,-3) \cup (-3,\infty)$

$$\frac{x - x^3 - 3x^2 + 9x + 27}{x+3}$$

$$f(x)-g(x) \; \boxed{\frac{-x^3-3x^2+10x+27}{x+3}}$$

Given: $f(x) = \dfrac{x}{x+3}$; $g(x) = x^2-9$

$$f(x) \cdot g(x) = \left(\frac{x}{x+3}\right)\left(\frac{x^2-9}{1}\right)$$

D: $(-\infty,-3) \cup (-3,\infty)$ $= \dfrac{x(x^2-9)}{x+3}$

$$= \frac{x(x-3)\cancel{(x+3)}}{\cancel{x+3}}$$

$$f(x) \cdot g(x) = x^2 - 3x$$

$$\frac{f(x)}{g(x)} = \frac{\frac{x}{x+3}}{x^2-9}$$

$$= \frac{x}{x+3} \cdot \frac{1}{x^2-9} \qquad x \neq -3$$

$x \neq 3$ or -3

$x^2-9=0$

$x^2=9$

$$\frac{f(x)}{g(x)} \; \boxed{= \frac{x}{x^3+3x^2-9x-27}}$$

$-\infty \leftarrow \; \times_{-3} \; \times_{3} \; \rightarrow \infty$

D: $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

Compositions of functions

- performing a function within a function

Notation

$(f \circ g)(x) \iff f(g(x))$

Ex: $f(x) = x+4$ $\quad g(x) = x^2 - 7$

$f(g(x)) = f(x^2-7)$

$= (x^2-7)+4$

$f(g(x)) \boxed{= x^2 - 3}$ D: $(-\infty, \infty)$

$(g \circ f)(x) = (x+4)^2 - 7$

$(x+4)(x+4) - 7$

$x^2+8x+16 - 7$

$\mathbb{R}$

D: $(-\infty, \infty)$ $\boxed{= x^2+8x+9}$

$f(x) = \sqrt{x-4}$ $\quad g(x) = 2x-5$

$g(f(x)) = \boxed{2\sqrt{x-4} - 5}$

D: $x-4 \geq 0$

$x \geq 4$

$[4, \infty)$

$f(g(x)) = \sqrt{2x-5-4}$

$\boxed{= \sqrt{2x-9}}$

D: $2x-9 \geq 0$

$2x \geq 9$

$x \geq 9/2$ or 4.5

$[9/2, \infty)$

Combinations: combining functions using the basic operations
$(+, -, \cdot, \div)$

Symbolic Rep given $f(x)$ & $g(x)$

$$f(x)+g(x) = (f+g)(x)$$
$$f(x)-g(x) = (f-g)(x)$$
$$f(x)\cdot g(x) = (f\cdot g)(x)$$

Domain is the same

$$f(x)\div g(x) = \left(\frac{f}{g}\right)(x)$$

Given: $f(x) = \dfrac{x}{x+3}$ $g(x) = x^2-9$

$$f(x)+g(x) = \frac{x}{x+3} + \frac{(x^2-9)}{1}\frac{(x+3)}{(x+3)}$$

$(x^2-9)(x+3)$

$x^3+3x^2-9x-27$

$$= \frac{\boxed{x}+x^3+3x^2\boxed{-9x}-27}{x+3}$$

New function $= \dfrac{x^3+3x^2-8x-27}{x+3}$ D: $x \neq -3$

$(-\infty,-3)\cup(-3,\infty)$

$$f(x)-g(x) = \frac{x}{x+3} - \frac{(x^2-9)}{1}\frac{(x+3)}{x+3}$$

$$= \frac{x-(x^3+3x^2-9x-27)}{x+3}$$

$$= \frac{\boxed{x}-x^3-3x^2\boxed{+9x}+27}{x+3}$$

New function $= \dfrac{-x^3-3x^2+10x+27}{x+3}$

D: $(-\infty,-3)\cup(-3,\infty)$

$f(x)\cdot g(x)$

$$\frac{x}{x+3}\cdot\frac{x^2-9}{1} = \frac{x(x^2-9)}{x+3}$$

x^2-9

$(x+3)(x-3)$

$$\frac{x\cancel{(x+3)}(x-3)}{\cancel{x+3}}$$

*Domain: $x \neq -3$

$(-\infty,-3)\cup(-3,\infty)$

$= x(x-3)$

or

x^2-3x