§ Arithmetic Sequence & Series

- a sequence is a function whose domain is the set of natural #'s.
- the terms of a sequence are the range elements of the function.
- An arithmetic sequence is a sequence in which there is a common constant being added between successive terms.
  - the constant being added in an arithmetic seq. is known as the common difference. (denoted d)

* Example: $-5, -2, 1, \ldots$

- looking at the seq, you are adding 3 to each # to get the next term
- therefore the common difference (d) is 3
- the next three terms would be 4, 7, 10

* With an Arithmetic Seq, it is also possible to find specific terms further in the sequence.

The n^{th} term of an Arithmetic Seq

$$a_n = a_1 + (n-1)d$$

- a_n: the actual term
- a_1: the 1st term of Sequence
- n: what # term
- d: Common difference

Arithmetic cont...

ex: $a_n = a_1 + (n-1)d$

given: 2, 6, 10, ... find the 20th term

$a_1 = 2 \quad d = 4 \quad n = 20$

$\therefore \quad a_{20} = 2 + (20-1)4$

$a_{20} = 2 + 19(4)$

$a_{20} = 78$, the 20th term would be 78

ex: Now given $a_{19} = 42$, $d = \frac{-2}{3}$, find the 1st term

$a_{19} = 42 \quad n = 19 \quad d = \frac{-2}{3}$

$\therefore \quad 42 = a_1 + (19-1)\frac{-2}{3}$

$42 = a_1 + (18)\frac{-2}{3}$

$42 = a_1 - 12$

$54 = a_1 \quad \therefore$ the 1st term is 54

Arithmetic Cont...

- **Arithmetic Means** are the terms between any two non consecutive terms of an arithmetic sequence.

ex: ① 5, 16, 27 ; 16 is the arithmetic mean

② 4, 9, 14, 19 ; 9 & 14 would be two arithmetic means b/w 4 & 19

Ex: find one arithmetic mean between 12 & 24

thus 12 (1) — (1) 24 looking for one #

- want to find "d"
- subtract first # from last # $24 - 12 = 12$
- now divided the difference by # of commas (,) : $\frac{12}{2} = \boxed{6}$ this is the common difference of the seq

∴ seq is 12, 18, 24 (+6, +6)

Ex: find five arithmetic means between 5 & 35

thus 5, ___, ___, ___, ___, ___, 35

- $35 - 5 = 30$
- $\frac{30}{6} = 5$ is common difference

∴ 5, 10, 15, 20, 25, 30, 35 (+5 +5 +5 +5 +5 +5)

Arithmetic Cont...

- an **Arithmetic Series** is the sum of the terms of the sequence
- S_n refers to the n^{th} **partial sum**: meaning the sum of the first "n" terms.

Sum of finite Arithmetic Series $\Rightarrow$ $S_n = \frac{n}{2}(a_1 + a_n)$

- S_n: Sum of "n" terms
- n: # of terms
- a_1: value of 1st term
- a_n: value of last term

Ex: find the sum of the first 60 terms of

$9 + 14 + 19 + \ldots + 304$

$n = 60 \quad a_1 = 9 \quad a_{60} = 304$

$\therefore \quad S_{60} = \frac{60}{2}(9 + 304)$

$S_{60} = 30(313)$

$$\boxed{S_{60} = 9390}$$

Arithmetic Cont...

ex: find the sum of $-3, 1, 5, \ldots$ for the first 40 terms

- we know $n = 40$, $a_1 = -3$, but not a_{40}
- to find a_{40} use $a_n = a_1 + (n-1)d$

$$a_{40} = -3 + (40-1)4$$

$$a_{40} = -3 + 156$$

$$a_{40} = 153$$

now $S_{40} = \frac{n}{2}(a_1 + a_{40})$

$$S_{40} = \frac{40}{2}(-3 + 153)$$

$$S_{40} = 20(150)$$

$$S_{40} = 3{,}000$$